XX. *The Influence of Stress and Strain on the Physical Properties of Matter.*

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[Plate 43.]

PART I.—ELASTICITY (continued).

The Internal Friction of Metals.

Origin and Purpose of the Investigation.

I have already had the honour of presenting to the Royal Society* an account of researches carried on with a view of determining the effects of stress and strain on the elasticity and electrical resistance of metals. Side by side with these researches were conducted others on magnetic induction and thermo-electricity, and a very considerable amount of experimental detail was collected with reference to the two last physical properties of matter. The results of the last-mentioned investigations have, however, now remained unpublished for several years, for it seemed desirable that, previously to publication, certain phenomena should, if this were possible, have light thrown upon them by further experiments on elasticity. More especially was I anxious to examine into the causes of the loss of energy experienced by a wire when vibrating torsionally, for the interesting memoirs of G. Wiedemann† and D. E. Hughes led me to expect that my doing so would cause some insight to be gained regarding the above-mentioned phenomena. The results of these labours, which have now occupied almost the whole of my spare time for the last three years, I offer to the Society in the hope that they may prove as interesting to others as they are to myself.

Researches of Thomson *and* Wiedemann.

Under the title of "The Elasticity and Viscosity of Metals," Sir W. Thomson published a memoir,‡ the first portion of which deals with the loss of energy of a wire

* 'Phil. Trans.,' Part 1, 1883.

† 'Wiedemann's Annalen,' vol. 6, 1879.

‡ 'Roy. Soc. Proc.,' vol. 14, 1865, p. 289.

when vibrating under the influence of torsional elasticity. It is pointed out (*a*) that, though no change of volume or shape can be produced without dissipation of energy, because of the accompanying change of temperature, estimates founded on the thermodynamic theory of elastic solids suffice to prove that the loss of energy due to this cause is small in comparison with the whole loss of energy which has been observed in many cases of vibration. (*b*) That, as a result of experiments in which a spring was vibrated alternately in air of ordinary pressure and in the exhausted receiver of an air-pump, there is an internal resistance to its motions immensely greater than the resistance of the air. Hence it is concluded that with *solids as with liquids* there exists an internal resistance to change of shape *depending upon the rapidity of the change.* The results of THOMSON's experiments are briefly as follows :—

(*a.*) The loss of energy in a vibration through one range was greater, the greater the velocity ;* but the difference between the losses at low and high speeds was much less than it would have been, had the resistance been approximately as the rapidity of the change of shape.

(*b.*) When the weight of the vibrator was increased, whilst its moment of inertia was maintained constant, the viscosity was always at first much increased ; but then, day after day, it gradually diminished and became as small in amount as it had been with the lighter weight.

(*c.*) A wire which was kept vibrating nearly all day, from day to day, after several days showed very much more molecular friction than another kept quiescent except during each experiment.†

The investigation was continued with much smaller degrees of maximum angular distortion, to discover, if possible, the law of the molecular friction, and, so far as the irregularities depending upon previous conditions of the elastic substance allowed any simple law to be indicated, it was proved that, as with fluids, the diminution of range per equal numbers of oscillations bore a constant ratio to the diminishing range. But, on the contrary, as with the larger angular distortions, the relation between the law of subsidence in two sets of oscillations having different periods, with the same elastic body in the same circumstances, was not that which would occur if the molecular resistance were simply proportional to the velocity of the change of shape in the different cases. If the molecular friction followed this simple law, the proportionate diminution of range per period would be inversely as the periods.‡ This proportion was not found to hold good, but the loss of energy was, in fact, as it would be if the result were wholly or partially due to the "elastische Nachwirkung"—elastic afterworking—as the Germans call it.

* I shall be able to show that this is not the case if we eliminate the effect of the resistance of the air.

† This so-called "fatigue of elasticity" does not occur if the oscillations are kept well within the limits of elasticity.—H. T.

‡ There is a slip in § 36 of 'Encycl. Brit.,' 9th edit., Art. "Elasticity," in which article Sir WILLIAM THOMSON refers to his previously-mentioned memoir ; the words "directly as the square roots of the periods" should be "inversely as the periods."

GUSTAV WIEDEMANN, at the end of a very interesting paper on "Torsion,"* discusses the question of the loss of energy experienced by a wire vibrating torsionally. He had by previous statical experiments proved—

(*a*.) That permanent torsion occurs even after the slightest temporary torsion.

(*b*.) That when a wire is repeatedly twisted by a given torsional stress, first one way and then the other, the distance between the permanent positions of equilibrium on each side gradually diminishes to a minimum; the period during which this diminution is taking place is called "the accommodation period."

(*c*.) That when the wire has been "accommodated" by stresses $\pm W_m$ repeatedly acting in opposite directions, till the temporary† and permanent torsions $\pm T_m$ and $\pm P_m$ respectively are constant, if the wire be again twisted by increasing stress in the direction of the last torsion, $T_p - P_p$ increases nearly in proportion to W_p, where T_p, P_p, and W_p represent any temporary torsion, permanent torsion, and torsional stress respectively between 0 and T_m, P_m, and W_m.

WIEDEMANN, from these and other results, feels himself justified in concluding that "the hypothesis according to which the diminution of the amplitude is to be referred to an internal friction which for the entire course of the oscillations is a function of the velocity can no longer be maintained." Neither does he consider that the elastic after-action will alone account for what is observed. The chief cause of the diminution of the amplitude lies rather, according to him, in the *shifting of the position of permanent torsion of the molecules at the end of each oscillation*; what takes place at the alternate oscillations may be well described in his own words as follows:—"In the absolute first position of rest of the wire, before any torsion in its molecules, we will draw axes parallel with the axis of the wire. Let the wire be next 'accommodated' by frequent vibrations to and fro, and at last be temporarily twisted in the positive direction, describing an angle of $+a$, while the molecules may be rotated so that the lower ends of their axes, looked at from the axis of the wire, describe an angle, $+\alpha$ say, to the left. If the wire be slowly brought back into the permanent torsion-position $+b$, the axes of the molecules will retain a portion $+\beta$ of their rotation to the left. If the wire now receive an impulse in the positive direction, which again sends it to $+a$, it will, according to the laws of perfect elasticity, swing back to $+b$. If it now swing beyond this position further to the right, and if the molecules in their rotation had no friction to overcome, it would arrive at the position $-a$, while the axes of the molecules would be rotated to $-\alpha$. Again, with perfect elasticity the wire would go back to the position $-b$, in which the molecules would be rotated $-\beta$, and so forth. The to-and-fro motions of the wire between $\pm a$ and $\pm b$ are perfectly elastic; therefore the performances of work in the swingings backward and forward

* 'Wiedemann's Annalen,' vol. 6, 1879, pp. 485–520; 'Phil. Mag.,' Jan. and Feb., 1880.

† WIEDEMANN employs the term "temporary torsion" so as to include not only the strain which disappears on the removal of the stress, but also the permanent torsion. I, on the contrary, will always denote the torsion which disappears on the removal of the torsional stress as the temporary torsion.

must within these limits completely compensate one another. In fact, however, there results a diminution of the amplitude of oscillation; hence the loss of energy therein can only correspond to the work which is expended in the alteration of the positions of equilibrium, or the rotation of the molecules from $+\beta$ to $-\beta$ which determine it. The diminution of amplitude may therefore be taken as a measure of this work. Since the shifting of the permanent torsion-position is, within certain narrow limits, proportional to the temporary torsion, and therefore the rotation of the molecules therewith is likewise approximately so, the amplitudes must, within those limits, diminish according to the law of a geometrical series."

Again, Professor WIEDEMANN agrees with Sir W. THOMSON that, whilst an increase of load at first increases the loss of energy, this loss, under repeated oscillations, becomes less and less, and finally approaches in magnitude that obtained with the lighter load. WIEDEMANN'S explanation of the matter is that, just as in his own statical experiments the torsional displacements were found to be initially greater with a heavier load than with a lighter one, so also must the decrements attendant upon oscillations be greater in the former case, but, like those, gradually approach the magnitude which is observed with the lighter loading.

Briefly expressed, then, the views of Sir W. THOMSON and Professor WIEDEMANN are as follows:—The former, whilst acknowledging that his experiments do not show that the proportionate loss of energy of a wire when vibrating torsionally is inversely as the period of vibration, which should be the case if the internal friction of a solid resemble that of a fluid, still employs the term *viscosity* with reference to metals, and appears to think that the internal friction of a metal partakes of the nature of the viscosity of a fluid; the latter is of opinion that the loss of energy is mainly due to a shifting from side to side of the positions of stable equilibrium of the molecules caused by the torsional vibrations of the wire.

I now proceed to give an account of my own researches relating to the internal friction of metals.

Description of Apparatus and Mode of Experimenting.

The wire under examination was firmly clamped at its upper extremity to a stout brass block, which was itself clamped to the end of a strong iron bracket projecting from the outside of the wall at the back of my house, the height of the bracket above the ground being 685·5 centims. A wooden box A A, nearly 600 centims. in length and 12 centims. square inside, protected the vertically suspended wire from currents of air, and passed through the roof of a small wooden house. The box was secured at its upper extremity to the iron bracket, and rested with its base B B upon another box S S (fig. 1), which measured about 40 centims. in length and breadth and about 86 centims. in height. The box S S had in front a glass window W W and a door, and at each of the two sides a door.* All the doors were lined with caoutchouc, so

* These doors are not shown in the figure.

as to fit air-tight, and were only opened for the purpose of adjustment. The top of the long box A A was also closed by a metal cap, and every precaution taken to ensure that the vibrations of the wire should be undisturbed by currents of air. The wire near its lower extremity passed through the centre of an aperture O made in the top of the box S S. The bar V V of the vibrator was clamped to the lower extremity of the wire, and to it were suspended two cylinders of equal mass and dimensions; by sliding the cylinders backwards or forwards on the bar, the moment of inertia of the whole vibrator could be altered without changing its mass. The bar V V was graduated in millimetres, and the cylinders could be clamped at any required equal distances from the axis of the wire; the position of the cylinders could readily be estimated to the tenth of a millimetre. The mode of attaching the cylinders is sufficiently shown in the figure, and it will be noticed that each cylinder is capable of being divided into two equal parts, each part having a mass of 337·1 grms., a length of 30·195 centims., and a diameter of 1·29864 centim. These cylinders, either complete or in halves, were used for nearly all the wires, except those of tin and lead, with which last lighter cylinders and bars were employed. The bar V V consisted in some experiments of a piece of thin, drawn, hollow brass tubing, of which the length was 30·70 centims. and the diameter 1·420 centims. Two brass caps provided with screws, about 8 centims. in length and 2 millims. in diameter, fit one into each end of this hollow bar, and can be easily removed from or placed in it. This bar will be designated as the bar B_2. In other experiments V V consisted of a thin hollow brass tube, closed at both ends, of length 30·46 centims., and, except at the centre, of mean diameter 0·94980 centim. The central portion of the bar had soldered on to it a solid concentric brass ring 1·55 centims. in length and of the same diameter as the bar B_1, so that either bar might be fitted into the brass block K, which was itself clamped to the wire. The bar last described will be designated as the bar B_1; both B_1 and B_2 can just pass through a circular aperture in the block K and be clamped at their centre to K. When the cylinders suspended to V V are used complete they will be designated as the cylinders C_2, and when they are used in halves as the cylinders C_1.

The torsional vibrations of the wire were observed by the usual mirror-and-scale arrangement, which is sufficiently shown in fig. 1, where M is the light mirror reflecting an illuminated circle of light crossed by a vertical, fine, dark line on to a scale placed at the distance of 1 metre from the mirror. The scale was bent into an arc of a circle 1 metre in radius, and was supported on brickwork, which last, together with the box S S, rested on the flagstones which formed the floor of the small wooden house previously mentioned. So perfectly firm and free from disturbing causes of whatever sort was the whole arrangement that the spot of light, when at rest, would remain perfectly so for any required length of time.*

* This, of course, shows that the arrangements made for securing freedom from currents of air answered their purpose satisfactorily.

Three thermometers, one near the top of the long box, one in the middle of the latter, and one in the box S S, served to determine the mean temperature of the wire; these thermometers will be designated as the thermometers T_1, T_2, and T_3 respectively. The thermometer T_3 was divided so as to read to one-tenth of a degree Centigrade, and had been tested recently at Kew; the thermometers T_1 and T_2 were carefully compared by myself with T_3 degree by degree, and were graduated in degrees Centigrade.

A delicate aneroid barometer, placed on a level with the middle of the box S S, served to determine the pressure of the air in the box; the aneroid had been recently compared with a standard mercury barometer, and was graduated to one-hundredth of an inch. Readings of the three thermometers T_1, T_2, and T_3, and of the aneroid, were taken at regular intervals during the experiments.

When all the adjustments had been completed, the wire, whether in the annealed or hard-drawn condition, was allowed to rest for a period of time varying, with the nature of the metal, from one day to several weeks, until there was no shifting of the position of torsional equilibrium and no appreciable "running down" of the wire either before or after vibration. It was necessary to adopt this precaution inasmuch as, though great care had been used in adjusting the wires to avoid imparting permanent or rather sub-permanent torsion, such was frequently found to exist, and manifested itself by a gradual shifting with time of the position of torsional equilibrium. Now, when a wire is either possessed of sub-permanent torsion or is "running down," or even on the point of doing so, the loss of energy which occurs during torsional vibration is appreciably greater than when there are no such things. Even when there may be no apparent permanent torsion, or "running down," it is necessary to allow the wire to rest for some time, and to cause it to undergo frequent vibrations before the internal friction assumes constancy.

When it was necessary to excite vibrations a side-door of the box S S was opened, and slight impulses were given by means of a worsted thread to the bar V V until the amplitude of the oscillations was of the required magnitude; the door was then shut, and another door* at the bottom of the long box A A was opened, so that any accidental pendulous oscillations might be checked: finally this door was closed, and when the amplitude of the oscillations had become reduced to the requisite extent the observer commenced to write down the extreme limits of several consecutive vibrations, as shown by the numbers on the scale. At intervals the time of transit of the light across the middle point of the scale was noted. When the amplitudes decreased rapidly the observations were continued throughout the experiment; but when the decrement was small the observer frequently left the room till the amplitude was sufficiently reduced, when he again noted down the extreme limits of the same number of consecutive vibrations as before, and so on. In all cases the experi-

* Not shown in the figure.

ments were repeated again and again until the logarithmic decrement became sensibly constant in the different experiments.*

In reducing the observations, the following plan was adopted in those cases where the decrement of the amplitude was comparatively slight. Suppose that a_1, b_1; a_2, b_2; a_3, b_3, and a_4 are seven consecutive readings,† the six corresponding arcs from rest to rest will be a_1+b_1, b_1+a_2, a_2+b_2, b_2+a_3, a_3+b_3, b_3+a_4. The means of a_1+b_1, b_3+a_4; b_1+a_2, a_3+b_3; a_2+b_2, b_2+a_3, were written down, and if these agreed well with each other, which was almost invariably the case, the logarithm of the total mean was taken. Now, say that n single vibrations have taken place between the end of this and the beginning of the next seven consecutive readings, the difference of the logarithms of the first and second total means will, when divided by $n+6$, give the mean logarithmic decrement for a single vibration. When the decrements of amplitude were comparatively large, the following mode of reduction was used :—Suppose a_1, b_1; a_2, b_2; a_3, b_3, and a_4 to be, as before, seven consecutive readings, so that the arcs from rest to rest are a_1+b_1, b_1+a_2, a_2+b_2, b_2+a_3, a_3+b_3, and b_3+a_4: the difference of the logarithms of a_1+b_1 and b_2+a_3; of b_1+a_2 and a_3+b_3; of a_2+b_2 and b_3+a_4, were put down, and, if they agreed well, the mean of these differences, divided by three, was taken as the mean logarithmic decrement of a single vibration. The following experiments will furnish illustrations of the methods pursued in the two cases, and will also give a fair idea of the accuracy to be attained.

* This is very necessary, as sometimes, from slight fluctuations of temperature, the internal friction is very variable. I have not unfrequently had to go on experimenting for a couple of hours or more before the logarithmic decrement became constant.

† In the earlier experiments seven readings were taken; in the later ones eleven readings.

Experiment I.

A WIRE of pure copper, hard-drawn, 602·5 centims. in length and ·009383 square centim. in section, allowed to rest for 24 hours after adjustment. Time of a single vibration, 13·930 seconds.

Number of arc.	Observed arc in scale-divisions.	Pairs of arcs between which means are taken.	Mean of pairs.	Total mean.	Probable error of total mean.	Logarithmic decrement for one vibration.	Departure of logarithmic decrement for one vibration from the mean.
1 2 3 4 5 6	281·9 280·3 280·4 279·8 277·9 277·3	1—6 2—5 3—4	279·60 279·10 280·10	279·60	·06		
71 72 73 74 75 76	233·0 232·5 232·0 231·0 230·9 230·5	71—76 72—75 73—74	231·75 231·70 231·50	231·65	·04	·001167	·000000
141 142 143 144 145 146	193·8 192·3 191·8 191·8 191·7 190·8	141—146 142—145 143—144	192·30 192·00 191·80	192·03	·09	·001164	—·000003
211 212 213 214 215 216	159·6 159·6 159·6 158·7 158·6 158·7	211—216 212—215 213—214	159·15 159·10 159·15	159·14	·01	·001166	—·000001
421 422 423 424 425 426	91·0 90·5 90·3 90·2 90·2 90·0	421—426 422—425 423—424	90·50 90·35 90·25	90·36	·06	·001170	+·000003
Mean	..	..	..	..	..	·001167	..

Experiment II.

A TIN wire, as pure as could be obtained by the ordinary process of distillation, 607·3 centims. in length and ·04972 centim. in diameter, tested about two months after suspension, having been in the meantime vibrated almost every day. Time of a single vibration, 3·898 seconds.

Number of arc.	Observed arc in scale-divisions.	Pairs between which the differences of logarithms are taken.	Difference of logarithms.	Logarithmic decrement for three vibrations.*
1	170·2	1—4	·06926	
2	160·2			
3	150·1	2—5	·07396	·07190
4	145·1			
5	135·0	3—6	·07246	
6	127·0			
7	123·1	7—10	·05550	
8	116·0			
9	112·1	8—11	·06016	·06392
10	108·0			
11	101·0	9—12	·07611	
12	94·1			
13	90·0	13—16	·06779	
14	85·1			
15	81·0	14—17	·06613	·06578
16	77·0			
17	73·0	15—18	·06343	
18	70·0			
19	68·0	19—22	·06907	
20	67·0			
21	64·0	20—23	·09375	·08160
22	58·0			
23	54·0	21—24	·08197	
24	53·0			
25	51·0	25—28	·04481	
26	49·0			
27	47·0	26—29	·04675	·05387
28	46·0			
29	44·0	27—30	·07004	
30	40·0			
31	38·0	31—34	·06127	
32	37·0			
33	35·0	32—35	·07684	·06835
34	33·0			
35	31·0	33—36	·06695	
36	30·0			
Mean	..	..	..	·06757

* The great variations which occur in the numbers in this column I afterwards found to be entirely due to the fact that in this case the period of torsional vibration was nearly synchronous with the period of pendulous vibration. (See note on pp. 809, 810.)

Remarks on Experiments I. and II.

It is worth while to pause here and to consider what we can glean by a careful examination of these two experiments, because they are typical of most of the others, whose number now can be reckoned by hundreds. I had in my mind rather a strong conviction that there was an intimate relationship between the loss of energy from internal molecular friction and the electrical resistance of a metal, and have already observed to that effect.* Accordingly, regarding this apparent relationship as a matter of considerable importance, I spared no pains in attempting to get accurate values for the coefficients of viscosity of metals, such as have been obtained by MAXWELL and others in the case of fluids. All the wires mentioned in this paper have been examined over and over again; nearly all have been set up a second time, and most of them a third time, and, whilst in no case has a wire been under examination for less than a week, in some instances the experiments have extended over several weeks. As the work progressed, the conviction became more and more forced upon me that I was dealing with something very distinctly different from what is met with in fluid friction. In Experiment I. we see apparently that one of the laws of fluid friction is *exactly* carried out, for according to the seventh column, in which is given the logarithmic decrement for one vibration, the diminution of range per equal numbers of oscillations bears a constant ratio to the diminishing range, the departures from the mean value of the logarithmic decrements being in no case greater than ·3 per cent. A closer scrutiny, however, shows us that this is only the case because, in calculating each of the logarithmic decrements, the mean of a great number of oscillations is taken.† If we look into each set of six arcs, we may notice that in some instances two consecutive arcs will show no diminution whatever, whilst in others the proportionate diminution is much more than the average. These are not the results of errors of observation, because the differences are appreciably greater than any which could be accounted for in this way, and when we turn to Experiment II. the phenomenon is more marked; we have most indubitable evidence of a continued rise and fall of the logarithmic decrement. This rise and fall would probably escape notice, were we able to take the mean of as many oscillations as were taken with copper; moreover, if we commence a fresh experiment altogether, we find a total mean logarithmic decrement agreeing, within the limits of errors of observation, with the previous one.

[Note added Sept. 18, 1886.—I have since discovered the cause of this rise and fall: it is due to the fact that it is impossible to ensure that the axis of rotation always passes *exactly* through the centre of mass. Of course, if the wire were *perfectly* flexible, and we could *entirely* avoid pendulous vibrations when we started

* 'Phil. Trans.,' Part 1, 1883, p. 168.

† It will be observed that in Experiment I. the mean logarithmic decrement for 70 oscillations is taken in all the series except the last, where we have the mean for 210 oscillations given.

the torsional vibrations, this would not be the case; but, as a matter of fact, even if we could *start* the wire without any pendulous motion, the axis of the wire never *exactly* passes through the centre of mass, and, as a consequence, when the wire vibrates torsionally, so-called "centrifugal force" causes torsional motion to be partially converted into pendulous motion, or *vice versâ*.

When, as in Experiment I., the period of pendulous vibration does not approximately synchronise with the period of torsional vibration, the phenomenon is not very pronounced; but when, as in Experiment II., the two periods approach to synchronism, it is very apparent. Indeed, when the two periods approach sufficiently to synchronism, and the damping is not great, the phenomenon may become so marked that, after a time, almost the whole of the torsional motion becomes converted into pendulous motion, and presently reconverted again, so that the torsional vibrations, having first diminished to nearly zero, begin to increase again until nearly the initial amplitude has been reached, when once more the torsional motion begins to be converted into pendulous motion, and so on.* An interesting fact in connexion with this phenomenon is that, if we allow the wire to rest, with oscillations at intervals, the rise and fall becomes much less pronounced, in consequence, probably, of the wire gradually allowing its axis to pass more nearly through the centre of mass, but we can easily reproduce it in all its intensity by jarring the wire,† or even sometimes by change of temperature.

Unless there is nearly synchronism between the pendulous and torsional vibrations, there is little or no *appreciable* effect on the value of the logarithmic decrement, and, if we reckon from maximum to maximum, I have found that the logarithmic decrement is *sensibly constant, whatever the amplitude,* provided the latter does not exceed a certain limit. Similarly there will be little error introduced in our estimation of the internal friction of the metal from the mean logarithmic decrement deduced from a number of trials, which number was in my experiments very considerable.

When, however, the synchronism between the two periods is sufficient, there may be a very serious interference with the *mean* logarithmic decrement, and even a sensible interference with the torsional vibration-period; and I hold it as a matter of some considerable importance that, in all experiments connected with the damping, or even, when great accuracy is required, with the vibration-period of torsionally vibrating bodies, this point should be well looked to.‡]

* I had the pleasure of showing this phenomenon to the Physical Society, June 26, 1886.

† A very slight shock sometimes will effect this.

‡ In the case of a magnet suspended with its axis horizontal, the axis of rotation will *of necessity* not pass through the centre of mass anywhere except at the magnetic equator.

Formula for determining the Loss of Energy experienced by a Metal Wire vibrating under the Influence of Torsional Elasticity and encountering a Resistance proportional to the Velocity.

The diminution of amplitude of a wire vibrating as in the present experiments is due to three causes :—

1. Energy imparted to the support to which the upper extremity of the wire is attached, or to the parts of the vibrator which may be capable of independent motion.*
2. Energy imparted to the air surrounding the wire and vibrator.
3. Internal molecular friction.

Of these causes, 1 may be dismissed at once as not affecting the results within the limits of errors of observation, and a means of eliminating the effect produced by the resistance of the air will be shown presently. I propose, therefore, in dealing with the problem before us, to confine myself to the consideration of Cause 3, on the assumption that the internal friction varies as the first power of the velocity.

Let F be the moment of the couple necessary to twist the wire through one radian ; $2k$ the resistance due to internal friction, when the wire is twisting or untwisting with unit angular velocity; τ the period of vibration from rest to rest ; and M the moment of inertia : then it can readily be shown that very approximately

$$\tau = \pi \sqrt{\frac{\mathrm{M}}{\mathrm{F}}}\left(1+\frac{k^2}{2\mathrm{FM}}\right), \qquad (1)$$

and that the proportionate diminution of amplitude is

$$\frac{k\pi^2}{\tau \mathrm{F}}. \qquad (2)$$

From (2) it follows that the logarithmic decrement should be independent of the amplitude of vibration, and should vary inversely as the period of vibration.

Discussion of THOMSON'S *Views concerning the internal Friction of Metals.*

According to the mathematical formula just given, the logarithmic decrement should, with the same vibration-period, be a constant for all amplitudes if the friction between the molecules of a metal resemble fluid friction. This we have seen is the case.† When I turned, however, to examine the effect of changing the moment of inertia of

* If, for example, the cylinders attached to the bar of the vibrator be capable of a motion independent of the bar, as they would be if suspended by fine wires or threads.

† Except, of course, in such cases as that of Experiment II., where the synchronism between torsional and pendulous vibration-periods causes the logarithmic decrement to vary ; or, rather, prevents there being any such thing as a logarithmic decrement.

the vibrator without at the same time altering the mass in order to prove the second law deduced above, I failed entirely, and, though I adopted numerous changes of the mass of the vibrator, of the amplitude of vibration, of the vibration-period, and of the nature of the metal, in no case was I able to get a step further towards proving the above-mentioned law. On the contrary, when I had eliminated the effect of the resistance of the air in the manner now to be described, I found the logarithmic decrement nearly independent of the vibration-period in some cases and increasing with it in others.

Mode of eliminating the Effect of the Resistance of the Air.

In a paper quite recently presented to the Royal Society, I have shown that the following formulæ for finding the logarithmic decrement due to the resistance of the air may be deduced from Prof. G. G. STOKES's valuable memoir "On the Effect of the Internal Friction of Fluids on the Motion of Pendulums":—*

For a cylinder vibrating horizontally

$$\log_{10} \text{dec.} = \frac{\pi^2 k' \rho a^2 l^3}{24\text{I}} \log_{10} e, \quad \ldots \ldots \ldots \quad (3)$$

where ρ is the density of the air, a the radius of the cylinder, l the length of the cylinder, I the moment of inertia of the vibrator, and k' is a constant, provided the vibration-period, the diameter of the cylinder, and the nature of the fluid remain unchanged.

For a pair of cylinders of equal mass and dimensions, suspended vertically at equal distances from the axis of the wire,

$$\log_{10} \text{dec.} = \frac{\pi^2 \rho \beta^2 l d^2 k'}{16\text{I}} \log_{10} e, \quad \ldots \ldots \ldots \quad (4)$$

where β is the diameter and l the length of each cylinder, and d is the distance between the two axes of the cylinders.

We further have from Prof. STOKES's paper the formula

$$\mathfrak{m} = \frac{\beta}{4} \sqrt{\frac{\pi \rho}{\tau . \mu}}, \quad \ldots \ldots \ldots \quad (5)$$

where τ is the vibration-period and μ is the coefficient of viscosity of air. When $\mathfrak{m}$ has been determined, k' can also be determined by the aid of the Table given on p. 46 of Prof. STOKES's paper.

By several very carefully executed series of experiments with vibrating cylinders

* (1850) 'Camb. Phil. Soc. Trans.,' vol. 9 No. X.

and spheres I have proved, in the paper just alluded to, that the coefficient of viscosity of ordinary air is given by the formula

$$\mu = \cdot 00017155(1 + 0 \cdot 002751t - \cdot 00000034t^2),^* \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where t is the temperature of the air in degrees Centigrade and the units C.G.S. units. From these four formulæ we can readily determine the logarithmic decrement due to the resistance of the air on the horizontal bar V V and on the two vertically suspended cylinders. The mode of proceeding is as follows :—First, from equation (6) is found the value of μ, which enables m to be obtained from equation (5). Then we obtain by interpolation, from the Table given on p. 46 of Prof. STOKES's paper, the value of k', and, by substituting this last in equations (3) or (4), according as we are determining the effect of the resistance of the air on the bar V V or on the two vertical cylinders, the logarithmic decrement required. The vertical cylinders are connected to the bar V V by the two suspenders x, x, the cylindrical portions of which are affected by the air to an extent which must be calculated independently of the long cylinders, which they connect with V V.† The effect of the resistance of the air on the other portions of the suspenders was determined experimentally by using the bar B_1 for V V, and observing the logarithmic decrement, first when the bar was vibrating with the suspenders attached, and afterwards with the suspenders removed, but with brass cylinders introduced into the hollow bar V V, of equal mass with the suspenders, and so adjusted that the moment of inertia remained unaltered.‡ By this means it was ascertained that the portions of the suspenders which were not cylindrical were equivalent to a certain length of the long vertical cylinders. This equivalent length would vary so slightly with the vibration-period that we may without sensible error consider it a constant; accordingly, in order to allow for this portion of the suspenders, the length of each of the long vertical cylinders was estimated at so much more.

* For the coefficients of t and t^2 in this formula I have taken as my authority HOLMAN ('Phil. Mag.,' vol. 21, 1886, p. 199), as not only do Prof. HOLMAN'S investigations seem to have been very carefully conducted, and to agree as regards the effect of change of temperature on the coefficient of viscosity with those of MEYER, PULUJ, and others, but also they are more in accordance with my own in this respect than are those of MAXWELL, who found the coefficient to vary as the absolute temperature.

MEYER and PULUJ have obtained values for the absolute coefficient of viscosity of air which differ from my own, but I prefer to use my own results, as these were obtained by experimenting with cylinders and bars of somewhat similar dimensions to those used here, and also under similar conditions to those holding here. I believe the equations given above to be capable of expressing the logarithmic decrement due to the resistance of the air with certainly a less error than 1 per cent.

† Because k' varies for cylinders of different diameters.

‡ For this purpose the caps were removed from the ends of B_1, and the brass cylinders, by means of companion-screws cut along their axes, were adjusted at the required distance on the screws attached to the caps; the caps were then replaced.

Equation (4) does not take into account the effect of the air due to the rotation of the cylinders about their axes; the correction required for this is, however, very slight.* When necessary, this correction was made by means of the following formula: †

$$\lambda_a = \frac{2M'\mu\tau}{I\rho} P \log_{10} e, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where λ_a is the natural logarithmic decrement due to the rotation of *both* cylinders about their axes, M′ is the mass of fluid displaced by *each* cylinder, and

$$P = \frac{f}{\sqrt{2}} + 1{\cdot}5 + \frac{0{\cdot}375}{\sqrt{2}f} - \frac{0{\cdot}4922}{\sqrt{2}f^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

f being put, by way of abbreviation, for $\sqrt{\frac{\pi\rho}{\mu\tau}}.a$, where a is the radius of the cylinder.

The Extent to which the Internal Friction of Metals depends upon the Vibration-Period.

We are now in a position to examine how far the loss of energy, resulting from internal friction, depends upon the vibration-period of the wire. I propose, therefore, to give, in illustration, the results of four sets of experiments on piano-steel, on copper, on zinc, and on tin.

Experiment III.

UNANNEALED Piano-Steel 0·0824 centim. in diameter and 602 centims. in length. Bar B_1 and Cylinders C_2.

Vibration-period in seconds.	Observed logarithmic decrement for one vibration.	Logarithmic decrement due to the resistance of the air.‡	Logarithmic decrement due to internal friction.	Temperature in degrees Centigrade.
3·9337	·0010497	·0006950	·0003547	10·00
5·5210	·0009563	·0006459	·0003104	10·64
8·1750	·0009633	·0005918	·0003715	10·12
12·5130	·0010710	·0007077	·0003633	9·70
16·9720	·0012052	·0008502	·0003550	11·38
Mean	..	..	·0003510	10·37

* Would not in any case exceed ½ per cent. of the whole.

† Kindly furnished by Prof. G. G. STOKES.

‡ It will be observed that the numbers in this column at first decrease with the time of swing and then increase. This arises from the fact that the time of swing, as well as the dimensions of the vibrator, come into the equations used for determining the logarithmic decrement due to the resistance of the air.

Experiment IV.

ANNEALED Copper Wire ·1622 centim. in diameter and 602 centims. in length. Bar B_2 and Cylinders C_2.

Vibration-period in seconds.	Observed logarithmic decrement for one vibration.	Logarithmic decrement due to the resistance of the air.	Logarithmic decrement due to internal molecular friction.	Temperature in degrees Centigrade.
1·7180	·0006597	·0003833	·0002764	17·13
2·2984	·0005971	·0003644	·0002327	15·20
3·1190	·0006136	·0003481	·0002655	15·17
4·7230	·0006552	·0003776	·0002776	14·60
6·3010	·0006835	·0004222	·0002613	17·90
7·9335	·0007347	·0004752	·0002595	16·20
Mean	..	..	·0002622	16·03

Experiment V.

UNANNEALED Zinc Wire ·07913 centim. in diameter and 602 centims. in length. Bar B_1 and Cylinders C_1 used.

Vibration-period in seconds.	Observed logarithmic decrement for one vibration.	Logarithmic decrement due to the resistance of the air.	Logarithmic decrement due to internal molecular friction.	Temperature in degrees Centigrade.*
5·875	·02116	·00113	·02003	19·86
7·222	·02243	·00106	·02137	19·86
15·313	·02632	·00094	·02538	19·86

Experiment VI.

A wire of tin, unannealed, 607·3 centims. in length and ·09944 centim. in diameter. The vibrator consisted of a solid cylindrical bar 23·3 centims. in length, 0·320 centim. in diameter, and 16·282 grms. in mass; suspended from the bar were two brass cylinders, each 1·127 centim. in diameter, 1·85 centim. in height, and 16·006 grms. in mass. The total mass of the vibrator was 49 grms.

* The temperatures varied from 18°·34 C. to 21°·37 C., but data were at hand for correcting the value of the logarithmic decrement when the temperature differed from the average, 19°·86 C. A similar remark applies to the temperatures 10°·25 C. and 3°·40 C. of the next experiment.

Vibration-period in seconds.	Observed logarithmic decrement for one vibration.	Logarithmic decrement due to the resistance of the air.	Logarithmic decrement due to internal molecular friction.	Temperature in degrees Centigrade.
1·579	·020236	·000913	·019323	10·25
2·362	·022346	·000831	·021515	10·25
3·898	·024343	·000912	·023431	10·25
The brass cylinders replaced by lead cylinders, so that the total mass of the vibrator was 168 grms.				
1·913	·018784	·000833	·017951	3·40
4·366	·023664	·000535	·023129	3·40
7·989	·024885	·000616	·024239	3·40
1·913	·020533	·000833	·019700	10·25
4·366	·026035	·000535	·025500	10·25
7·989	·027316	·000616	·026700	10·25

Remarks on Experiments III.–VI. inclusive.

Let us now see what we can gather from the data given in these experiments. With tin the logarithmic decrement decidedly increases with the vibration-period, but not in the same proportion as the latter, and may be expressed by the following formula :—

$$\lambda_\tau = a + b\tau + c\tau^2,$$

where τ is the vibration-period and λ_τ the logarithmic decrement, whilst a, b, and c are constants, the last being a negative quantity. Thus, when the mass of the vibrator in Experiment VI. was 49 grms., and the temperature 10°·25 C., the formula became

$$\lambda_\tau = \cdot012407 + \cdot005437\tau - \cdot0006693\tau^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

whilst for the same temperature, and with the mass of the vibrator equal to 168 grms., it was

$$\lambda_\tau = \cdot012381 + \cdot004466\tau - \cdot0003347\tau^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The logarithmic decrement would thus seem to be capable of being divided into two parts, the one being independent of the load and of the vibration-period, and the other dependent upon the load and the vibration-period.

When the mass of the vibrator was 168 grms., and the temperature 3°·40 C., the formula became

$$\lambda_\tau = \cdot011470 + \cdot0039473\,\tau - \cdot0002925\,\tau^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Now, if we allow for the difference of temperature, the value of a given in (11) will be found to accord almost exactly with the values of a given in (9) and (10).

The effect of change of temperature is very considerable, for a rise of 6°·85 C. caused an increase of 10·05 per cent. in the internal friction, and proved to be nearly the same for the three vibration-periods: 1·913 second, 4·366 seconds, and 7·989 seconds.

With zinc the logarithmic decrement also increases with the vibration-period, but not to the same extent as with tin, the formula in this case becoming

$$\lambda_\tau = \cdot 01019 + \cdot 0022274\,\tau - \cdot 0000941\tau^2. \quad \ldots \ldots \quad (12)$$

With the metals possessing less internal friction, steel and annealed copper, the variation of the logarithmic decrement with the period of vibration is so slight as to be masked by errors due to small fluctuations of temperature* and other causes; and, had not a great many other observations than those here recorded proved that the internal friction does slightly increase with the vibration-period, it would have almost appeared not to do so. If, however, we compare in Experiments III. and IV. the *mean* value of the first half of the logarithmic decrements with that of the second half, we may trace indications of the truth of the above statement. To sum up, then, these and many other experiments of a like kind have nearly satisfied me that, with *all* metals, the logarithmic decrement does increase with the period of vibration, but that, the less internal friction the metal possesses, the more nearly is the former independent of the latter.

Effect of Fluctuations of Temperature on the Internal Friction.

Perhaps one of the most remarkable features presented by a torsionally vibrating wire is the ease with which the molecules are disturbed by changes of temperature from those positions of stable equilibrium into which they will, after a period of time, whose length depends upon the nature of the metal, naturally creep. It is truly astonishing to observe the profound change in the internal molecular friction of such an elastic metal as unannealed piano-steel† which is wrought by a sudden rise or fall of one or two degrees of temperature. I have explained that, though the wires were shielded by a long box, this box was itself exposed, for the greater part of its length, to the external air. Now, the first wire examined was one of unannealed piano-steel, and some hundreds of trials were made with it during a period of three months, and

* See what follows shortly.

† I do not know that unannealed piano-steel is more remarkable than other metals in the respect mentioned.—H. T.

at various hours of the day. Sometimes the proportionate diminution of amplitude would seem quite as constant for different ranges of amplitude as could be expected, but at others there would be variations far exceeding anything which could be attributed to errors of observation. For some weeks I was quite at a loss to account for these variations, but at length was successful. Luckily, I had determined to make frequent observations of all three thermometers, and, whilst examining the contents of my note-book, it occurred to me that the greatest variations took place on those days and hours at which the variations of temperature were greatest. I also noticed that *the logarithmic decrement became larger, whether the change of temperature was of the nature of a rise or a fall.* At first this explanation seemed almost incredible, for the variations of temperature would sometimes be only 1° C., or even less, and yet at the same time the value of the logarithmic decrement would be increased 30, 40, or even 100 per cent. My suspicions having been thus roused, I found it comparatively easy to verify them, and among several other experiments, made with the same object, tried the following :—

Experiment VII.

The same piano-steel wire as in Experiment III. The sun was shining rather faintly, and at intervals on that side of the house on which the wire was suspended, but before the end of the experiment passed over to the other side.

Civil time.	Temperature of the wire in degrees Centigrade.	Alteration of temperature in degrees per hour.	Time of 100 vibrations in seconds.	Logarithmic decrement due to internal friction.
h. m. s.			m. s.	
10 10 42 A.M.	10·33	..	..	..
10 40 57	9·33	1·98−	30+13·05	·000636
11 11 10	10·00	1·32+	+14·35	·000635
11 41 24	11·00	1·98+	+12·80	·000753
12 18 39 P.M.	10·23	..	..	vibrator restarted
12 48 52	10·43	0·396+	+12·90	·000428
1 49 15	10·23	0·198−	+11·08	·000356
2 49 37	10·10	0·129−	+10·93	·000501
4 50 21	9·17	0·461−	+10·61	·000702

From 7 P.M. to 9 P.M., when the experiment terminated, the logarithmic decrement and the temperature became nearly constant, the value of the former being ·000356.

The changes of temperature, as shown in the second column, are very small, but we see what a large effect they have on the internal friction, which in one case became twice as great as when under a fairly steady temperature. The effects of fluctuations of temperature on the internal friction resemble those of mechanical jarrings, such as would be produced when the mass of the vibrator is altered, or even when the cylinders are shifted along the bar without any alteration of mass;* they also, as I shall show in a future paper, can be imitated by means of magnetic stress.

Attention should also be directed to the fourth column of the experiment, where it will be seen that not only is the loss of energy increased by change of temperature, but so also is the vibration-period; the latter is, however, so much less altered than the former that it requires careful observation to detect any such similarity between them.

In consequence of the above-mentioned effect of fluctuations of temperature on the internal friction, it was necessary to choose a day, or time of day, when the temperature was fairly steady, and all the results which will be recorded in the following portion of this memoir were obtained when this was the case, for with all the metals which I have examined the phenomenon is more or less pronounced.

Since it is impossible, with the arrangements which I have described, to obtain a *perfectly* steady temperature, it seems probable that most of the values of the logarithmic decrements previously recorded are slightly in excess of what they would have been, could we have secured greater uniformity in this respect.

The question next arises: will any change of temperature, *however small,* introduce increase of internal friction? I think not; for not only in some instances have I obtained values for the logarithmic decrements, which were as constant as the limits of errors of observation would allow, and yet where during the experiments the temperature did change several times, and in an irregular manner, though by small amounts, but I shall in another paper bring forward instances where the molecules were slightly twisted backwards and forwards by small magnetic stress without appreciably affecting the value of the logarithmic decrement.

The Effect of Loading on the Internal Friction.

We have seen that according to both Sir WILLIAM THOMSON and Professor G. WIEDEMANN, when the load on the wire is increased, there is at first an increase in the internal friction, but this increase diminishes with time, so that the logarithmic decrement tends to become the same as with the lighter load. Neither of the above observers has, however, attempted to ascertain whether or not the internal friction is *entirely* independent of the load, provided sufficient time be allowed, and sufficient account be taken of the effect of the resistance of the air in diminishing the amplitude of vibration. The following arrangements were therefore made for the purpose of settling this point by experiment:—

The brass block, into which was clamped the bar V V of the vibrator (fig. 2),

* This is one of the reasons why the logarithmic decrement is at first increased when the mass of the vibrator is decreased or increased, even though the change of mass produce no perceptible permanent alteration whatever in the length of the wire, and why it is prudent to allow the wire to oscillate for some time after shifting the cylinders, as in Experiments III., IV., V.

terminated at its lower extremity S in a ring, through which a pin *pp* could just pass. The pin *pp* served to connect the brass box A with the bar of the vibrator by passing through two rings *r*, *r*, attached to the top of A, and placed one on each side of the ring at S. In this way the box A was well secured to V V, and without the least chance of any shifting taking place, as the wire vibrated torsionally. Two smaller boxes B, B, were suspended from V V, and could be clamped, as shown in the figure, to any part of it. All three boxes were provided with hinged doors * of the same height as the boxes, so as to admit of the introduction of two sets of cylinders C and L, the one set of brass and the other of lead; these doors, after the cylinders had been arranged in the boxes, were kept shut by means of springs and clasps. The mass of each of the brass cylinders C was 100 grms., and that of each of the lead cylinders L 2000 grms. The masses of each of the cylinders in the two sets were adjusted by myself, so as to agree with some standard masses within 1 part in 10,000. Each of the cylinders L had been cast in the same mould, and the dimensions had been carefully measured by means of calipers reading to $\frac{1}{1000}$th inch. The moment of inertia of each cylinder was determined from the dimensions and mass, and afterwards by certain indirect methods, which last, when suitable precautions were taken, gave results in good accordance with those which had been obtained by the direct method. Moreover, the moments of inertia of all the lead cylinders were found to be very nearly equal to each other. The mode of proceeding was as follows:—Eight of the lead cylinders L were placed in the box A, and on the top of these was put a wooden cylinder W, fitting rather tightly in A. On the top of W, and fitting into a circular cavity let into the centre of it, were placed sixteen of the brass cylinders C, arranged in sets of four each, each set having its four constituents bound together by an elastic band *d*. The wooden cylinder W served not only to prevent any rocking of the lead cylinders as the wire vibrated, but also to enable the experimenter to readily adjust the brass cylinders C in their proper positions in the box A.

When all the necessary adjustments had been carefully completed, the whole was allowed to remain for five days at rest, except that the wire was caused to vibrate several hundred times on each day; finally, the logarithmic decrement and the vibration-period were determined from a very large number of oscillations. As soon as these had been determined in a sufficiently satisfactory manner, two of the lead cylinders L were removed from A, and four of the little brass cylinders C were transferred from A to the boxes B, B, two being placed in each box. The boxes B, B, had been at the outset so placed on V V that the above transference would exactly make up for the loss of the moment of inertia which would follow from the removal of the two lead cylinders. After this second adjustment and a rest of twenty-four hours, followed by a large number of preliminary vibrations, the logarithmic decrement and the vibration-period were again determined. Again, two more lead cylinders were

* Not shown in the figure.

removed from A and four more transferred from A to B,* and so on, until at length all the lead cylinders had been removed from A and all the brass cylinders transferred from A to the boxes B, B. The next experiments show the results obtained by treating steel and copper wires in this manner.

Experiment VIII.

Unannealed piano steel, 0·0824 centim. in diameter and 602 centims. in length. Mean vibration-period, 21·2825 seconds. The temperature ranged from 6° C. to 7° C.

Load on the wire in kilos.	Logarithmic decrement due to internal friction.
4·635	·0003633
8·635	·0003381
12·635	·0003479
16·635	·0003658
20·635	·0003547

Experiment IX.

Annealed copper wire, 0·1622 centim. in diameter and 602 centims. in length. Mean vibration-period, 7·340 seconds. The temperature ranged from 13° C. to 20° C.

Load on the wire in kilos.	Logarithmic decrement due to internal friction.
4·635	·0002653
8·635	·0002239
12·635	·0002610
16·635	·0002616
20·635	·0002991

The variations, which occur in the values of the logarithmic decrements for different loads, are to be attributed to differences in the rate at which the temperature varied on different days. Had a greater number of observations been made with each of the loads on *different* days, the mean values of the logarithmic decrements would have varied still less. We may say, then, that for comparatively large loads the internal friction is independent of the load, whether the wire be annealed or hard-drawn.

We will now turn to consider the effect of comparatively light loading on the internal friction.

* The figure shows the arrangements of the cylinders after this last operation.

Experiment X.

The piano-steel wire used in Experiment VII. was, after the removal of the boxes, tested with suspended cylinders of different masses. The temperature ranged from 10° C. to 15° C., and the vibration-period from 3·934 seconds to 4·608 seconds.

Total mass of the vibrator in grammes.	Logarithmic decrement due to internal friction.
164	·0003602
838	·0003763
1512	·0003547

Experiment XI.

The annealed copper wire of Experiment IX., after having been tested in the manner described in this experiment, was tried with much lighter loads. The temperature ranged from 17°·9 C. to 18° C., and the vibration-period from 6·301 seconds to 7·363 seconds.

Total mass of the vibrator in grammes.	Logarithmic decrement due to internal friction.
1570	·0002756
4635	·0002653

Experiments X. and XI. teach us that both for hard-drawn and annealed wires the internal friction is as independent of the load for light loads as we have seen it to be for heavy ones. We must, however, in such experiments as these be careful to begin with the heaviest load first, as loading tends more or less to produce a diminution of the internal friction, which is partly permanent and partly sub-permanent, provided no sensible permanent elongation results from the loading. Care should also be taken, in changing from the heavier load to the lighter one, to avoid shocks. If these precautions be not taken, the logarithmic decrement will probably be found less for the heavier load than the lighter one.

The Effect of Loading on the Torsional Elasticity.

It is, I believe, generally assumed that the torsional elasticity of a wire is independent of the load, but apparently no experimental evidence has been brought forward in support of this assumption, at any rate as far as large stresses are concerned.* We might, perhaps, be inclined to infer that, because the internal friction is independent of the load, so also will be the torsional elasticity, but my own experience had taught me that it was by no means safe to do so. It is true that Experiments VIII. and IX. had shown that the torsional elasticity must be nearly independent of the load when the latter is at all large, inasmuch as the period of vibration was nearly the same for all the loads. There were, however, slight differences in the vibration-period for the different loads, which I felt inclined to attribute to the fact that the boxes might not always hang quite vertically. The following experiments seem to show conclusively that for loads ranging from moderate amounts to great ones the period of vibration is independent of the load.

Experiment XII.

A pan made of brass was suspended by two hooks, H, H (fig. 3); the bottom of the pan was circular, and the two suspenders H A, H A, made of drawn brass, were soldered into A. The lead cylinders before-mentioned were in the first instance placed on the pan, so that their centres were in the same line as the axis of the wire. The brass cylinders, C_2, were placed near the ends of the bar V V, at equal distances from the wire, and had in consequence of their position and mass a very considerable moment of inertia as compared with the rest of the vibrator. After the arrangements had been completed the apparatus was allowed to remain suspended for several days, and was during this period frequently vibrated. The vibration-period was then determined, and, two of the lead cylinders having been removed, a further rest of 24 hours was given,† when again the vibration-period was determined; and so on, until all the lead cylinders had been removed. In this experiment the same piano-steel wire as before was used. The temperature ranged from 9°·30 C. to 13°·40 C.

Vibration-period in seconds.	Load on the wire in kilos.	Couple in dyne centims. required to twist the wire through one radian.
16·658	1·208	5713
18·106	5·208	5738
19·889	9·208	5758
21·331	13·208	5766
22·720	17·208	5754
24·003	21·208	5756

* G. WIEDEMANN has shown in his paper already quoted that for moderate loads the torsional elasticity is independent of the load.

† Rest should always be given either after putting on or taking off load, as, though the torsional elasticity is not so much affected by shocks as the internal friction, it is affected.

Experiment XIII.

The copper wire used in Experiment IX. was treated in the same manner as the steel in the last experiment, except that now the cylinders C_2 were dispensed with. The temperature throughout remained nearly constant at 20° C.

Vibration period in seconds.	Load on the wire in kilos.	Couple in dyne centims. required to twist the wire through one radian.
1·891	0·534	46539
3·327	4·534	46841
4·312	8·534	46818
5·105	12·534	46708
5·801	16·534	46817

Remarks on Experiments XII. and XIII.

Experiment XII. shows that for the unannealed piano-steel the couple required to twist the wire through one radian is quite independent of the load for loads ranging from 4 to 20 kilos. For smaller loads than 4 kilos. the value of the couple is slightly less. With the annealed copper, the couple required to twist the wire through one radian is independent of the load for all the loads.

Having thus endeavoured to prove that the torsional elasticity does not depend upon the load when the latter is comparatively large, both for hard-drawn and annealed wires, I will consider the effect of light loading. As regards annealed wire, Experiment XIII. furnishes evidence that the elasticity is very nearly independent of the load, even for light loads, for the lightest load in this experiment does not much exceed $\frac{1}{2}$ kilo.* Care must, however, be taken, with annealed wire, to begin with the heaviest load first, as loading, if not carried to the extent of causing perceptible permanent elongation, invariably produces an increase of elasticity, which is partly permanent and partly sub-permanent, and this last must not be disturbed by shocks in unloading if we are to strictly compare the values of the torsional elasticity with different loads.†

As regards the unannealed piano-steel, the case is otherwise, for if the loads be small the torsional elasticity may vary very perceptibly with them.

* This would produce only a very small stress per square centimetre as compared with the stress used with the unannealed piano-steel.

† See similar remarks respecting the internal friction. With the heavier loads still remaining on the wire, the sub-permanent elasticity is not so readily shaken out as with the lighter ones. I have since thoroughly satisfied myself that the torsional elasticity is independent of the load, even for light loads, in the case of annealed wires, provided sufficient time be allowed after changing each load.

Experiment XIV.

The same piano-steel wire as before. The temperature ranged from 9°·30 C. to 12°·38 C.

Vibration-period in seconds.	Total load on the wire in grammes.	Couple in dyne centims. required to twist the wire through one radian.
4·303	164	5496
16·658	1208	5713
18·106	5208	5738

It will be noticed that there is a comparatively rapid rise in the torsional elasticity as the load increases from 164 grammes to 1208 grammes, and a much less proportionate increase as the load rises still further. Now we find this phenomenon to be closely associated with the permanent torsion, which is, to a greater or less extent, always imparted during the process of wire-drawing. Permanent torsion produces decrease of torsional elasticity, but, as I have already shown,* unannealed piano-steel, if permanently twisted by hand, begins to temporarily untwist† when loaded, the amount of untwist being, for small loads, very rapid, and consequently we might perhaps expect the elasticity to be increased by loading, rapidly at first, and afterwards much more slowly as the loads were increased in amount. Similarly piano-steel behaves when permanently twisted by wire-drawing; this is apparent from the next experiment.‡

Experiment XV.

The unannealed piano-steel used in the previous experiments, and which had been, during a period of several weeks, frequently loaded and unloaded with weights reaching to upwards of 20 kilos., was allowed to rest for some time with only the bar of the vibrator § attached; a pan whose mass was 388·8 grammes was then suspended

* 'Phil. Trans.,' Part I., 1883, p. 21.

† This expression needs a little explanation. By the words "*permanently* twisted" is meant that the torsion will remain in the wire so long as we do not alter the load under the influence of which the wire was when the torsion was originally imparted, or, if we do alter the load, but afterwards restore it to the original amount, the twist originally imparted will be restored.

‡ I have since examined the matter more closely, and find that, with all very hard-drawn wires, there is a slight difference between the torsional elasticity, as inferred from torsional vibrations, when different light loads are used. This is also the case with annealed metals which, after annealing, have suffered considerable permanent torsion. Whether the torsional elasticity increases or diminishes with the load depends apparently upon whether loading causes the wire to temporarily untwist or twist more.

§ The mass of this bar was 164 grammes.

by means of hooks from the bar, and afterwards loaded carefully with weights, which eventually reached 16 kilos.

Load added to the bar in grammes.	Untwist in scale-divisions produced by each additional load.
388·8*	1814·0
1000	453·5
1000	113·8
2000	106·3
2000	78·3
2000	59·3
2000	43·8
2000	45·8
2000	39·0
2000	37·8

The Effect of Permanent Strain on the Internal Friction.

Experiment XVI.

An annealed iron wire, 602 centimetres in length and 0·960 centimetre in diameter, was provided with a vibrator whose mass was 3606 grammes; a rest of 24 hours was now allowed, and after this period the wire was tested, the results of the experiments showing a logarithmic decrement of ·001131. Next the wire was permanently elongated to the extent of 3·16 per cent., and, having been shortened to its original length, was again tested about ten minutes afterwards; the logarithmic decrement was now found to be ·001422. After a rest of 24 hours, the internal friction had so far diminished that the logarithmic decrement had become ·001214, *i.e.*, nearly 8 per cent. greater than it was before the permanent extension had taken place. The vibration-period before the permanent strain was 4·309 seconds, and after the extension 4·505 seconds; therefore, allowing for the diminution of section, resulting from the strain, the effect of the latter was to cause a decrease of the torsional rigidity of 2·8 per cent.

We must observe, with reference to such an experiment as the above, that, in order that permanent extension may produce increase of internal friction, the wire must be loaded, when tested before the extension, to a degree which, though not sufficient to cause sensible permanent elongation, is yet enough to secure a certain amount of permanent strain among the molecules; if the wire be only tested with a light vibrator, permanent extension may possibly produce a diminution of internal friction, as the following experiment shows.

* The load produced by the pan itself.

Experiment XVII.

An annealed copper wire 602 centims. in length and 0·960 centim. in diameter was provided with a vibrator whose mass was 430 grammes, and after resting for several days was tested; the logarithmic decrement proving to be ·000297. The wire was now permanently lengthened 2·67 per cent., and ten minutes after it had been shortened to its original length and re-suspended was again tested, with the following results:—

Logarithmic decrement due to internal friction.	Remarks.
·000717	Mean value for the first 100 vibrations
·000496	Mean value for the next 200 vibrations
After a rest of two days.	
·000262	Mean value for 400 vibrations

Thus we see that, though the permanent extension increased considerably the internal friction, when the wire was vibrated shortly after the strain had been produced, the effect was rapidly lessened by rest, so that after two days the logarithmic decrement became slightly less than it was before the extension. The time of vibration was, before the extension, 5·125 seconds, and after the extension 5·275 seconds. If we allow for the diminution of section, caused by the strain, there results from extension a *diminution** of the *torsional rigidity* of about 0·7 per cent.

Experiment XVIII.

The same wire was now twisted through twenty complete revolutions, of which six came out, leaving a permanent torsion of fourteen revolutions; ten minutes afterwards the wire, which had been kept vibrating, was again examined, with the following results:—

Time of vibration in seconds.	Logarithmic decrement due to internal friction.	Remarks.
5·475	·001175	Mean value for the first 100 vibrations
5·440	·001051	Mean value for the next 200 vibrations
After a rest of 24 hours.		
5·425	·000461	Mean value for 400 vibrations

* Observe that diminution of torsional rigidity is not always attended by increase of internal friction.

It would appear that permanent torsion has much more effect than permanent extension in increasing the internal friction. The torsional rigidity of the metal is also much more diminished by permanent torsion than by permanent extension, but the effect on the rigidity is nothing like so great as that on the internal friction, which last amounts to nearly 76 per cent.

Having shown that when sensible permanent deformation is produced, either by traction or torsion, there is an increase of internal friction, I will now consider the effect of stress, which is not sufficient to produce sensible molar permanent deformation, but which causes molecular deformation.

Experiment XIX.

An annealed iron wire 601 centims. in length and 0·960 centim. in diameter. Attached to the bar of the vibrator was a pan, which, together with the bar, weighed rather more than 2 kilos. The wire was, after adjustment, allowed to remain for five days, and was in the meantime frequently vibrated. After this period weights were put into the pan, and after each addition of load, followed by a rest of about ten minutes, the logarithmic decrement was determined; before each determination the wire was suffered to make about 100 vibrations.

Load in kilos.	Logarithmic decrement due to internal friction.	Remarks.
2	·003241	
4	·002726	
6	·002231	
8	·001406	
2	·001492	
4	·001246	
6	·001606	
8	·001346	
10	·002377	
12	·004177	
14	·005563	
14	·002441	After a rest of 30 minutes.
14	·002366	Started again.
14	·005606	After having been vibrated for about ten minutes through an angle of 60°.*
14	·001268	After a rest of 20 minutes.

An additional load of 1 kilo. was now put on and caused a slight permanent extension, which, after a rest of two days, was ascertained to be 2·6 centims. The wire was then reduced to its original length, and, 14 kilos. being the load, was allowed to vibrate for ten minutes. After this the logarithmic decrement was again determined.

* Notice the "fatigue of elasticity" (see p. 832).

Load in kilos.	Logarithmic decrement due to internal friction.	Remarks.
14	·001017	After a rest of 24 hours and a very large number of previous oscillations.
10	·001089	Ditto.
6	·001109	Ditto.
2	·001307	Ditto.

The above experiment suffices to show the great extent to which the internal friction can be diminished by the permanent molecular strain resulting from loads which are not sufficient to cause any appreciable permanent extension. For we see that even with a load of 15 kilos. the permanent extension was only about 0·4 per cent., whereas a load of half this was sufficient to cause a permanent molecular strain which reduced the friction to less than one-half of the original amount.

The Effect of Permanent Strain on the Torsional Rigidity.

I have already pointed out in Experiments XVI., XVII., and XVIII. that when very sensible permanent molar deformation is produced, either by traction or torsion, the torsional rigidity is decreased,* so that we need only now consider the effect of stress which, without producing sensible molar deformation, suffices to cause molecular permanent strain.

Experiment XX.

An annealed copper wire 602 centims. in length and 0·1622 centim. in diameter. The mass of the vibrator attached to the wire was 1554 grammes, and the determinations of the logarithmic decrements were all made with only the vibrator. When, however, it was necessary to strain the wire, a pan was suspended from the bar of the vibrator, and on this pan weights were placed which were suffered to remain for some time. The pan and weights were then removed, and 24 hours after the removal the testing, preceded by a large number of preliminary vibrations, began. The temperature ranged from 15° C. to 20° C.

Load in kilos. used to produce the permanent strain.	Number of days during which the load acted before removal.	Couple in dyne-centimetres required to twist the wire through one radian.
1·554	0	41477
1·554	8	44373
21·554	6	48158

* Sir W. THOMSON had previously shown this to be the case ('Encycl. Brit.,' 9th edit., Art. "Elasticity," § 81).

There was no perceptible permanent elongation produced by the loading, so that the increase of torsional elasticity, which is very apparent, must be due to molecular and not molar strain. Part of the strain is only sub-permanent and can be shaken out; when this is done there is an immediate loss of elasticity, which is regained if the wire be again loaded to the same extent as before, and subjected to the same previous treatment.

Effect of Wire-drawing on the Internal Friction.

We have seen that both permanent extension and permanent torsion are attended with permanent increase of internal friction. Now, in the process of wire-drawing there occurs both permanent extension and torsion, combined with lateral pressure. As a consequence, wire-drawing increases the internal friction in an astonishing manner. I shall have occasion to dwell more fully on the effects of annealing in a memoir which I hope will shortly follow this one, but the following Table will bear witness to the truth of the above statement:—

TABLE.

Name of metal.	Logarithmic decrement due to internal friction in the hard-drawn metal. A.	Logarithmic decrement due to internal friction in the annealed metal. B.	Ratio A : B.
Silver	·001540	·0002970	5·2
Copper	·000526	·0002622	2·0
Platinum	·002239	·0000766	29·3
Platinum-silver	·001792	·0001166	15·3
Zinc	·018530	·0032070	5·8

We thus see that annealing the hard-drawn metals reduced the internal friction to an amount which varied from *one-half to one-thirtieth of the original.*

The Logarithmic Decrement due to Internal Friction is independent of the Length and Diameter of the Wire.

Experiment XXI.

The annealed copper wire of Experiment IV. was drawn to a diameter which was nearly one-half of the original and afterwards re-annealed. With the same length of wire in both cases, and with nearly the same preliminary treatment, the logarithmic decrement due to internal friction only was for the thicker wire ·0002622 and for the thinner one ·0002695.* The temperature of the wire was nearly the same in each case,

* Great care must be used in an experiment of this kind to ensure uniformity of temperature during the period of experimenting, otherwise the thinner wire will show a greater internal friction than the thicker one (see Experiment VII.).

and the difference in the two logarithmic decrements is not greater than could be accounted for by slight difference in the steadiness of the temperature.* Two nickel wires, whose diameters were also nearly in the same ratio, namely 2 : 1, gave values for the logarithmic decrements due to internal friction which agreed with each other within the errors of observation.

Experiment XXII.

Two pieces of the same specimen of annealed zinc wire, one 602 centims. in length and the other only 97 centims. in length, were both tried under nearly the same conditions as regards preliminary treatment and time of vibration, &c. The logarithmic decrement due to internal friction was, in the case of the longer wire, ·003207, and, in the case of the shorter one, ·003327. The difference between these two numbers can quite be accounted for by the fact that the temperature of the shorter wire was about 3° C. higher than that of the longer.†

Two pieces of iron wire, cut from the same hank, but one having a length of 96 centims. and the other of 602 centims., gave, when treated in a similar manner, exactly the same logarithmic decrements.

We may, therefore, regard the logarithmic decrement as independent both of the length and of the diameter of the wire, provided care be taken that the two pieces of different lengths which are compared shall have suffered the same preliminary treatment.

"Fatigue of Elasticity."

Sir WILLIAM THOMSON writes,‡ "Experimental exercises performed by students in the physical laboratory of the University of Glasgow, during the session 1864–65, brought to light some very remarkable and interesting results, proving a loss of energy in elastic vibrators incomparably greater than anything which could be due to imperfections in their elasticity, and showing also a very remarkable fatigue of elasticity, according to which a wire, which had been kept vibrating for several hours or days through a certain range, came to rest much more quickly when left to itself than when set in vibration after it had been at rest for several days and then immediately left to itself." This so-called "fatigue of elasticity" seemed to me such a very remarkable phenomenon that, throughout the whole of these investigations, I was constantly on the look-out for traces of it; but, owing no doubt to the very slight maximum deformations employed in these experiments, I could detect nothing of the kind with any metal§ except nickel. The whole of the other metals examined, namely,

* It is necessary to take great care that the temperature should fluctuate very little, as any fluctuation will affect a thin wire more than a thicker one.

† The effect of change of temperature on the internal friction of zinc is rather considerable.

‡ 'Encycl. Brit.,' 9th edit., Art. "Elasticity," § 30.

§ Except in Experiment XIX.

platinum, silver, steel, iron, brass, copper, aluminium, zinc, platinum-silver, and German-silver, with arcs from rest to rest ranging up to 700 scale-divisions, and lead and tin, for arcs ranging up to 200 scale-divisions, showed no sign of the phenomenon. With nickel, however, the case was different, and I met with most marked signs of "fatigue of elasticity," even for such small molecular displacements as attended arcs ranging from 300 to 700 scale-divisions.

Experiment XXIII.

A nickel wire, unannealed, 602 centims., and ·018622 square centim. in section, after having been tested for several days with three different vibrators, was finally tried with the most massive of the three.*

Logarithmic decrement due to internal friction.	Vibration-period in seconds.	Remarks.
·007058	6·010	Immediately after adjustment.
·006623	..	After ten minutes' rest.
·006225	..	After $1\frac{1}{2}$ hours' further rest.
·005964	..	After 2 hours' further rest.
·005585	..	After $3\frac{1}{2}$ hours' further rest.
·005522	5·960	After 16 hours' further rest.
·005644	..	Started again immediately.
·005689	..	Ditto.
·006023	..	An arc of 700 scale-divisions maintained for 2 minutes, and then allowed to subside.
·006427	..	An arc of 700 scale-divisions maintained for 30 minutes, and then allowed to subside.

In all the above trials the initial arc from which the logarithmic decrements were computed was about 600 scale-divisions, and the final arc about 150 scale-divisions. I next give the whole of the observations which were taken in the last trial from the initial to the final arc.

Range of arc.	Logarithmic decrement due to internal friction.
605·2–516·0	·00674
516·0–440·8	·00665
440·8–372·0	·00718
372·0–317·2	·00673
317·2–270·0	·00680
270·0–230·8	·00663
230·8–201·5	·00570
201·5–174·0	·00619
174·0–152·5	·00553
152·5–135·5	·00495

* The total mass of this vibrator was 3666 grammes.

It will be noticed that the logarithmic decrement begins to fall very rapidly as soon as the range of arc has become 230·8—201·5. Immediately, therefore, that the calculations necessary for obtaining the above results had been finished, the vibrator was again started, but now with only an initial arc of about 300 scale-divisions.

Range of arc.	Logarithmic decrement due to internal friction.
299·2–165·8	·004938
165·8– 67·5	·003713
67·5– 34·8	·002687
34·8– 19·0	·002438
19·0– 11·2	·002106

By this time the vibration-period had diminished to 5·900 seconds.

After a rest of five hours the vibrator was started with an initial arc of just over 100 scale-divisions.

Range of arc.	Logarithmic decrement due to internal friction.
100·2–57·8	·002200
57·8–36·8	·001771
36·8–22 0	·002044
Mean	·002005

Remarks on Experiment XXIII.

In the first place, we may direct our attention to the diminution of the vibration-period produced by rest after repeated vibrations; this seems to be almost, if not quite, as pronounced as it is in the case of iron, the vibration-period gradually diminishing from 6·010 seconds to 5·900 seconds, and this, too, in spite of the fact that the same vibrator had been used several times previously with the wire, and had been carefully adjusted at the commencement of the experiment.*

In the second place may be noticed the diminution of the logarithmic decrement which attends the above-mentioned diminution of the vibration-period, and that the former of the two is very much greater than the latter. This last is easily accounted for if we assume with WIEDEMANN that the internal friction is due to the to-and-fro rotation of the molecules about their axes, and not, as is the torsional elasticity of the wire, mainly upon the force necessary to alter the distances separating the centres of the molecules.

Thirdly, the effect of "elastic fatigue" becomes less and less apparent as the ampli-

* It should be understood that the *bar* of the vibrator was not changed in any of the alterations of the mass of the vibrator, but only the cylinders.

tudes through which the logarithmic decrement is calculated diminish, and, moreover, the range through which the logarithmic decrement is a constant becomes wider as the amount of "elastic fatigue" is diminished. We may see here analogies between the increase of the proportionate diminution of amplitude caused by "elastic fatigue" and the increase of the ratio of the temporary molar deformation, whether torsional or longitudinal, to the stress producing it, as the latter increases whenever the wire acted upon has recently suffered permanent longitudinal extension or torsion.*

Lastly, it is desirable to draw attention to the marvellously small deformations which can produce "elastic fatigue" in nickel as compared with those which are attended with corresponding effects in the case of the other metals.

The Effect of Rest and of frequent Oscillations on the Internal Friction.

It has been already observed that rest after strain of any kind invariably diminishes the internal friction, but rest may be considerably aided in this effect by agitating the molecules either by thermal or by mechanical agency. With the part played by thermal agents I shall deal in a later memoir, and now turn to agitations produced by oscillating the wire torsionally.

Experiment XXIV.

The wire used in Experiment XVI., and which had suffered the permanent extension mentioned in this experiment, was allowed to rest for 24 hours with a load of 12 kilos. on, and was then tested with the same load still on.

Logarithmic decrement.†	Range of arc.	Number of oscillations before the logarithmic decrement was determined.
·003050	537·4–500·9	4
·003108	411·5–346·6	40
·002617	346·6–299·9	64
·002875	299·9–255·8	88
·002905	255·8–217·9	112
·002553	217·9–189·2	136
Started again with an initial arc of 600 scale-divisions, and left vibrating until the arc was reduced to 379·0 scale-divisions.		
·002033	379·0–302·7	
·001911	302·7–245·1	

* 'Phil. Trans.,' 1883, Part I., pp. 7, 8. See also WIEDEMANN's paper *loc. cit.*

† In this experiment the effect of the resistance of the air was not eliminated, as it was impossible to have done so accurately, and the elimination would have been of little importance.

The mean of the two last values of the logarithmic decrement is ·001972, and here it remained very fairly constant, further oscillation not diminishing the internal friction. When, however, rest for one or more days was now given, the internal friction would rise again until once more repeated oscillations would reduce it to ·001972 or thereabout. No doubt the increase of internal friction attending on rest in this case is owing to changes of temperature which occur in the interval, such changes, as we have seen, tending to disturb the molecular arrangements which existed at the end of the frequent oscillations.

Summary.

1. The proportionate diminution of amplitude due to internal molecular friction in a torsionally vibrating wire is independent of the amplitude, provided the deformations produced do not exceed a certain limit. This limit varies with the nature of the metal, and is, for nickel wire, very low.

2. The logarithmic decrement of amplitude increases with the length of the vibration-period, but in a less proportion than the latter, and in a diminishing ratio. The amount of increase of the logarithmic decrement attending on a given increase through a given range of the vibration-period varies with the nature of the metal, and with those metals in which the internal friction is small becomes nearly insensible.

3. It follows from 2 that the internal friction of a solid does not resemble the viscosity of fluids.

4. Permanent molecular strain resulting from loading, not carried to a sufficient extent to produce sensible permanent extension, diminishes the internal friction and increases the torsional elasticity.

5. Considerable permanent longitudinal extension and permanent torsion produce increase of internal friction and diminution of torsional elasticity. The effect of torsion is much greater than that of extension, and the increase of internal friction is much greater than the decrease of torsional elasticity.

6. The internal friction of a metal is independent of the temporary load.

7. Annealing a hard-drawn wire may enormously diminish the internal friction, especially if, after annealing, the wire be kept suspended for some days and frequently vibrated.

8. Rest after suspension, aided by not too large oscillations at intervals, diminishes the internal friction of a wire which has been recently suspended, or which, after suspension, has been subjected to considerable molecular agitations by mechanical or thermal agency.

9. "Fatigue of elasticity" is not felt, provided the deformations produced do not exceed a certain limit. This limit varies considerably with the nature of the metal,

being very low for nickel, so that with this metal it is difficult to avoid "elastic fatigue."

10. Mechanical shocks and rapid fluctuations of temperature beyond certain limits may increase very considerably the internal friction, and, though to a much less extent, diminish the torsional elasticity.

11. The logarithmic decrement is independent both of the length and of the diameter of the wire.

Fig. 2.

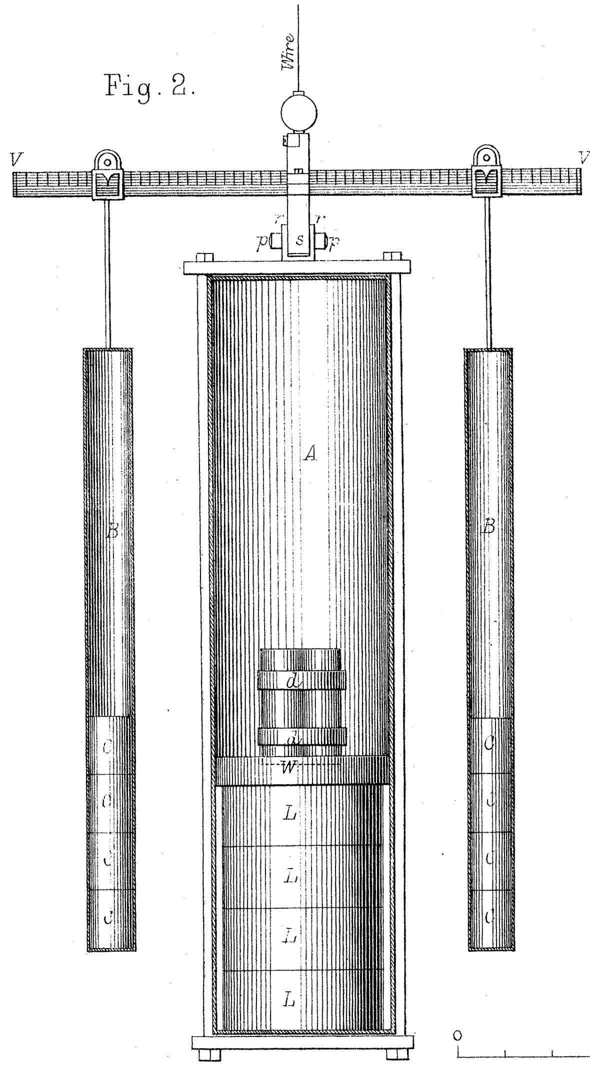
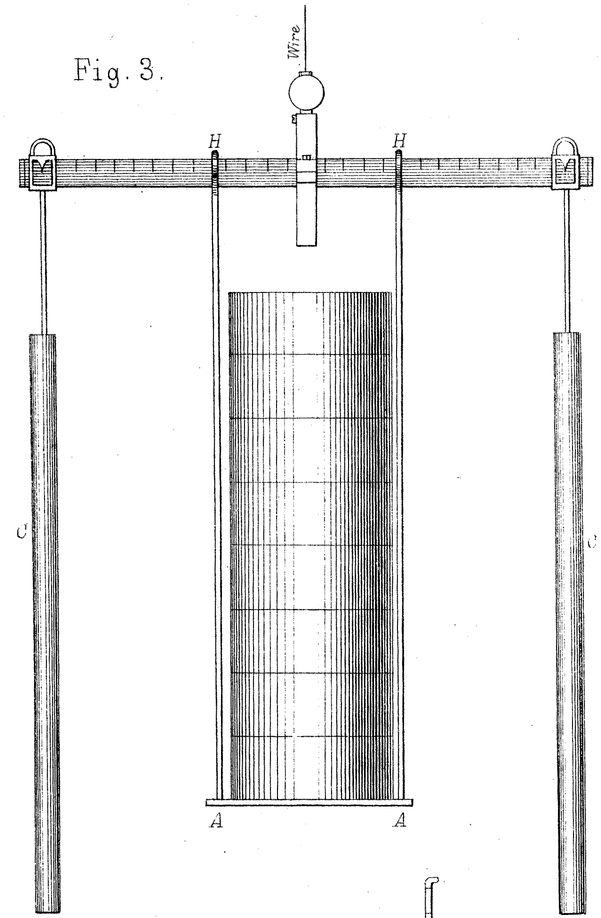


Fig. 3.



0 3 6 9 12 Ins.

Scale of Figs. 2 & 3.

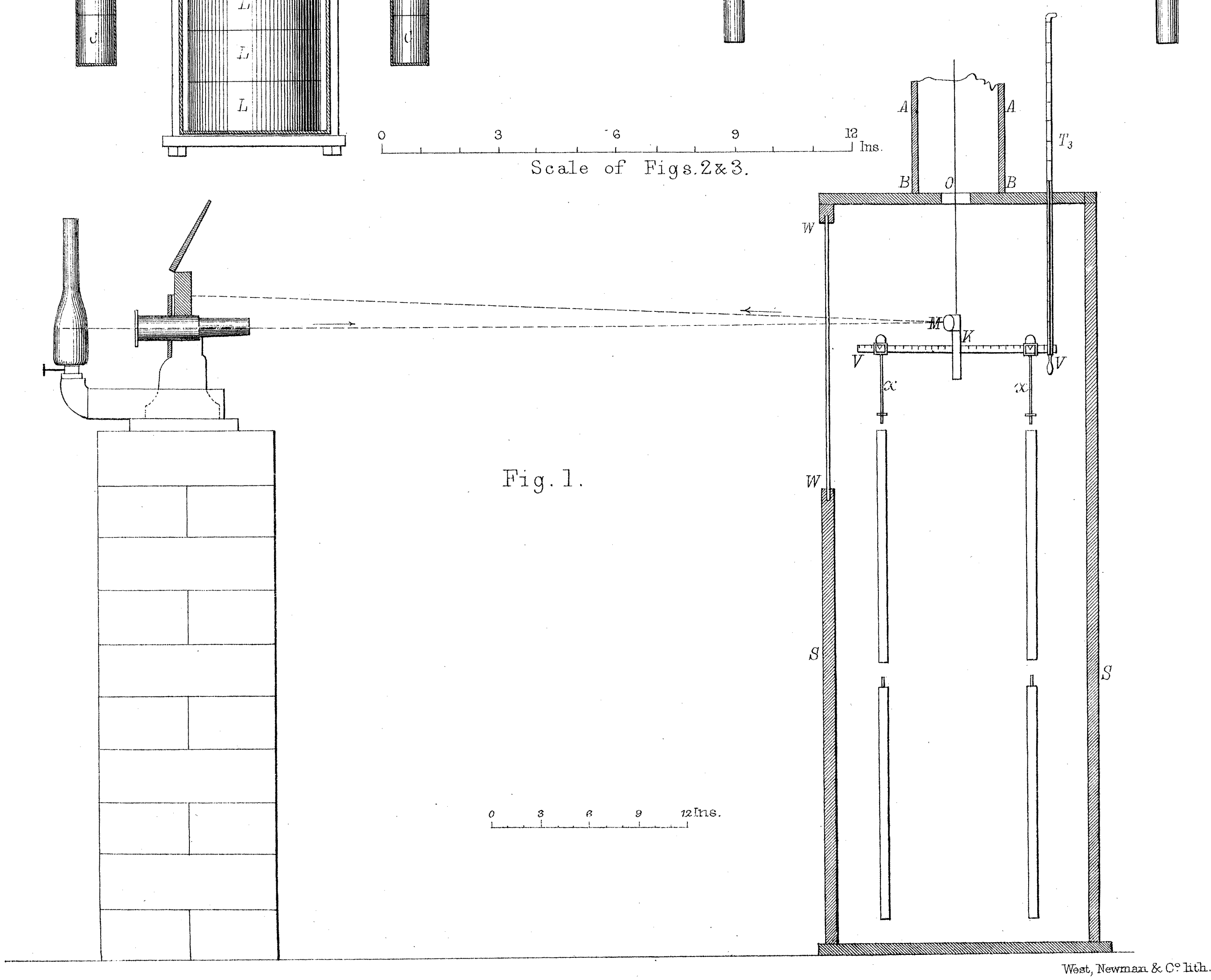


Fig. 1.

West, Newman & Co. lith.